数理逻辑（2025 春）作业 -07

1 [Enderton，第 79 页，第 (1,2,5) 题] 在英文与指定的一阶语言之间进行翻译。

- Any uninteresting number with the property that all smaller numbers are interesting certainly is interesting. ($\forall$, for all things; N, is a number; I, interesting; $<$, is less than; 0, a constant symbol intended to denote zero.)
- There is no number such that no number is less than it. (The same language as in Item a.)
- $\forall x(Nx \to Ix \to \neg\forall y(Ny \to Iy \to \neg x < y))$. (The same language as in Item a. There exists a relatively concise translation.)
- (I) You can fool some of the people all of the time. (II) You can fool all of the people some of the time. (III) You can't fool all of the people all of the time. ($\forall$, for all things; P, is a person; T , is a time; $F\,x\,y$, you can fool x at y. One or more of the above may be ambiguous, in which case you will need more than one translation.)

2 [Enderton，第 129 页，第 1 题]

对于一个项 u，记 u_t^x 为用项 t 代替变量 x 所得到的表达式。请在不使用“替换”或其同义词的前提下重新表述这个定义。

3 [Enderton，第 130 页，第 9 题]

- 用两个例子展示 $(\varphi_y^x)_x^y$ 通常不等于 φ，其中：
  - 第一个例子展示 x 可以出现在 $(\varphi_y^x)_x^y$ 中，但不出现在 φ 中；
  - 第二个例子展示 x 可以出现在 φ 中，但不出现在 $(\varphi_y^x)_x^y$ 中。
- 证明 **再代入引理（Re-replacement lemma）**：如果 y 不出现在 φ 中，则 x 可在 φ_y^x 中代入 y，并有 $(\varphi_y^x)_x^y = \varphi$。